Where does stuff come from?

Factors of production

Technology

Factors of production: “Thanks, Marx!”

The stuff we use to make other stuff

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Thanks, Karl Marx!

Returns to Scale

How does a change in the inputs affect the output?

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"Cobb-Douglas" production function:

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CRS: $\alpha + \beta = 1$
DRS: $\alpha + \beta < 1$
IRS: $\alpha + \beta > 1$

Returns to Scale

Expect CRS

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"Give me another Paris"

Returns to Scale

What if we only increase one input? $Y = K^{.3}L^{.7}$

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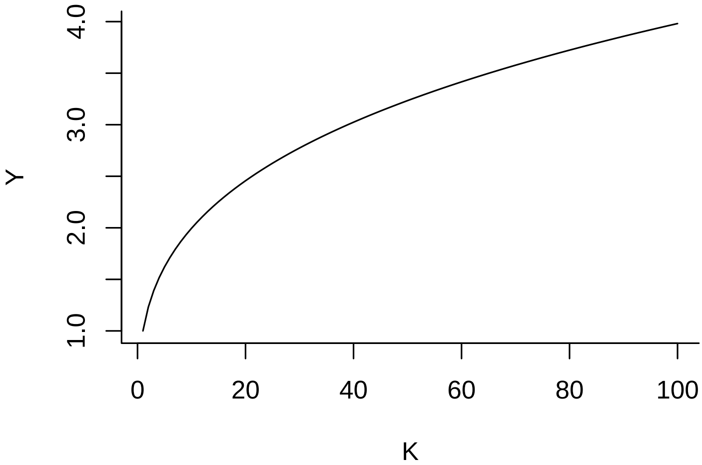
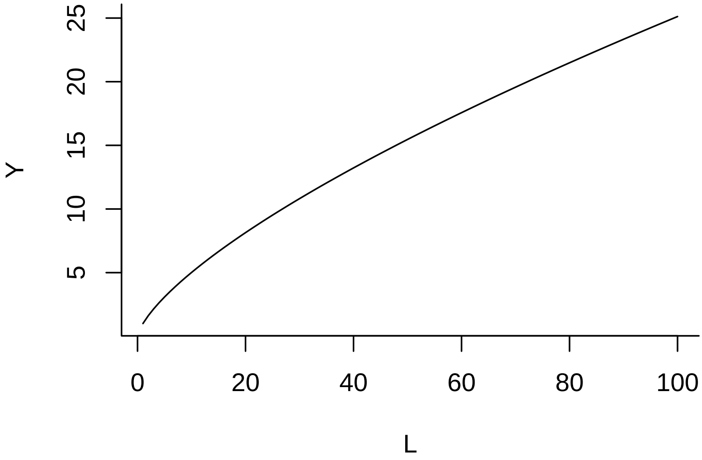


Returns to Scale

What if we only increase one input? $Y = K^{.3}L^{.7}$



What does this look like in 3d? $Y = K^{.3}L^{.7}$

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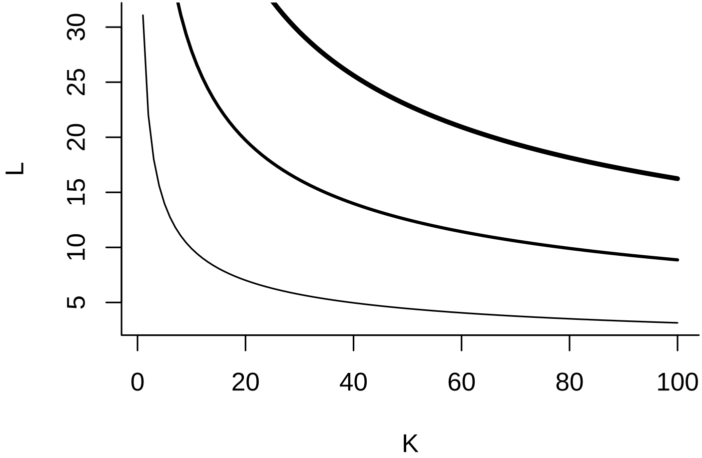
"Level curves":



Why .3 and .7?

These are the shares of total income (GDP) we observe going to each group of inputs.

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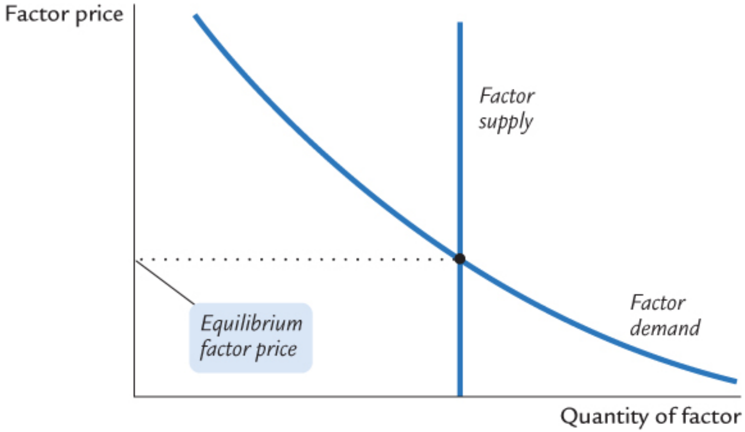
Determined by supply and demand for inputs and their prices:

$$
\begin{aligned}
w &= \text{labor wage} \\
r &= \text{capital rental rate}
\end{aligned}
$$

Determination of factor prices



The Firm's Problem

- Sell Y at price P
- Hire L at price w
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Profit maximization rule: MB = MC

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MB: extra revenue

MC: extra payments to inputs

Marginal product

What is the marginal benefit of hiring another worker?

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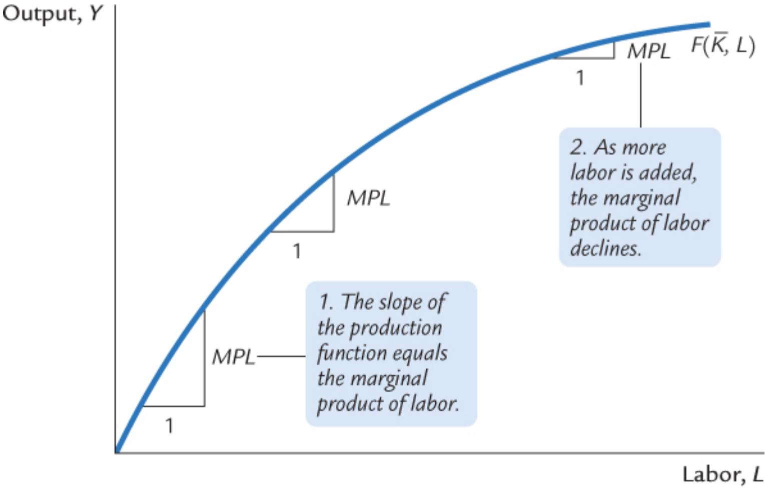
$$\text{MPL} = F\left(K, L+1\right) - F\left(K, L\right)$$

$$\text{MPL} = F'_L\left(K, L\right)$$

Marginal product of labor



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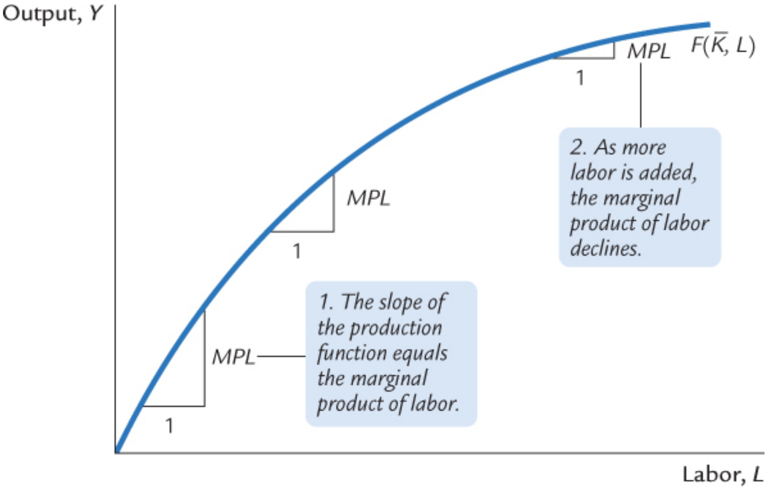

Why does the MPL fall?

Marginal Products with Cobb-Douglas

$$\text{MPL} = F'_L(K, L) = \beta K^{\alpha} L^{\beta - 1}$$

Marginal Products with Cobb-Douglas

$$\mathsf{MPL} = F'_L(K, L) = \beta K^\alpha L^{\beta-1}$$

$$\beta = 1 - \alpha \Rightarrow \begin{cases} \mathsf{MPL} = (1-\alpha)\left(\frac{K}{L}\right)^\alpha & = (1-\alpha)\frac{Y}{L} \\ \\ \mathsf{MPK} = \alpha\left(\frac{L}{K}\right)^{1-\alpha} & = \alpha\frac{Y}{K} \end{cases}$$

(Remember the power rule!)

$$\text{Profit} = \text{Revenue} - \text{Costs}$$

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Profit maximization rule:

$$\text{MR} = \text{MC} \Longleftrightarrow \begin{cases} P\text{MPL} = w \\ P\text{MPK} = r \end{cases} \Longleftrightarrow \begin{cases} \text{MPL} = \frac{w}{P} \\ \text{MPK} = \frac{r}{P} \end{cases}$$

Division of National Income

$$Y = L \times \text{MPL} + K \times \text{MPK} + Profits$$

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Cobb-Douglas example:

$$\begin{aligned} & L \times \text{MPL} + K \times \text{MPK} \\ & \qquad = L\left(1-\alpha\right)\left(\frac{K}{L}\right)^{\alpha} + K\left(\alpha\right)\left(\frac{L}{K}\right)^{1-\alpha} \\ & \qquad = \left(1-\alpha\right)K^{\alpha}L^{1-\alpha} + \alpha K^{\alpha}L^{1-\alpha} \\ & \qquad = K^{\alpha}L^{1-\alpha} \end{aligned}$$

Income Shares

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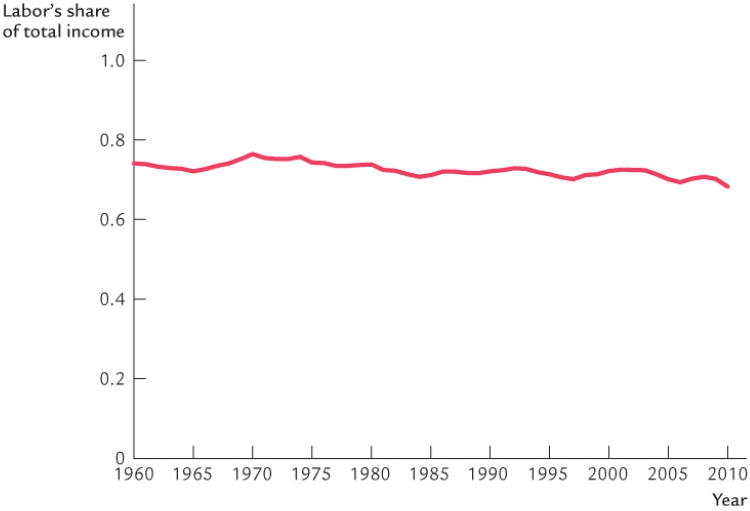
Proportion of total income going to labor: $\frac{wL}{Y} = 1 - \alpha$

Income Shares



Theory of Everything

The magical macro equation:

$$Y = AK^{.3}L^{.7}$$