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$$Y = AF(K, L) \tag{1}$$

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1. Get more A
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Size does not matter

$$y = f(k)$$

Returns to scale?

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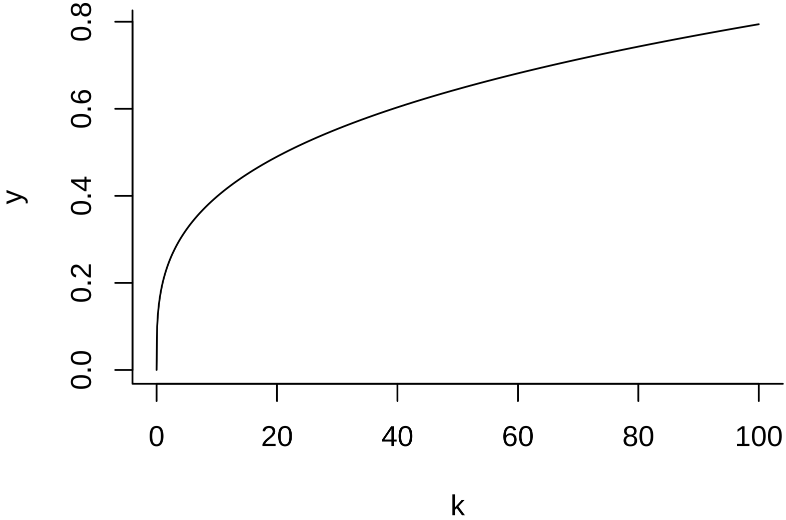
Returns to scale?

What happens to MPK as we get more capital?


0.0
0.2
0.4
0.6
0.8
y
0
20
40
60
80
100
k

Solow Model

Components of economic models

- Behavior
- Equilibrium
- Dynamics

Solow Model: Behavior

What do we do with our incomes?

$$y = c + i \tag{6}$$

Solow Model: Behavior

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Solow Model: Behavior

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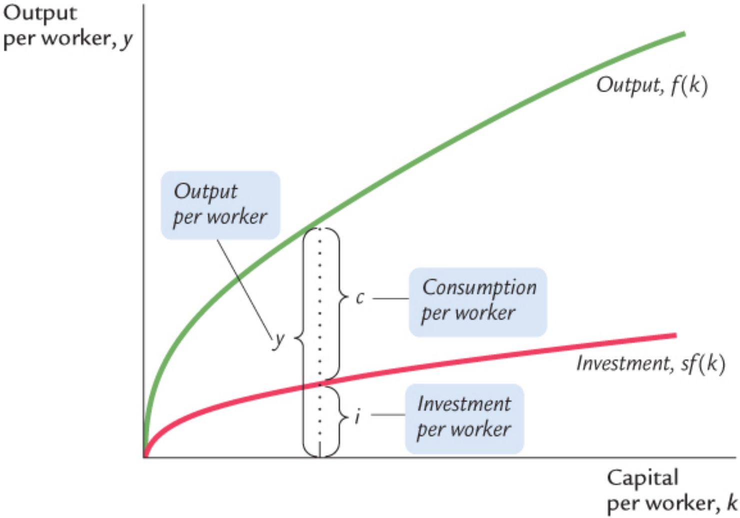
How much do we invest?
Suppose we save a constant fraction s ("exogenous")

$$c = (1 - s)\, y \tag{7}$$

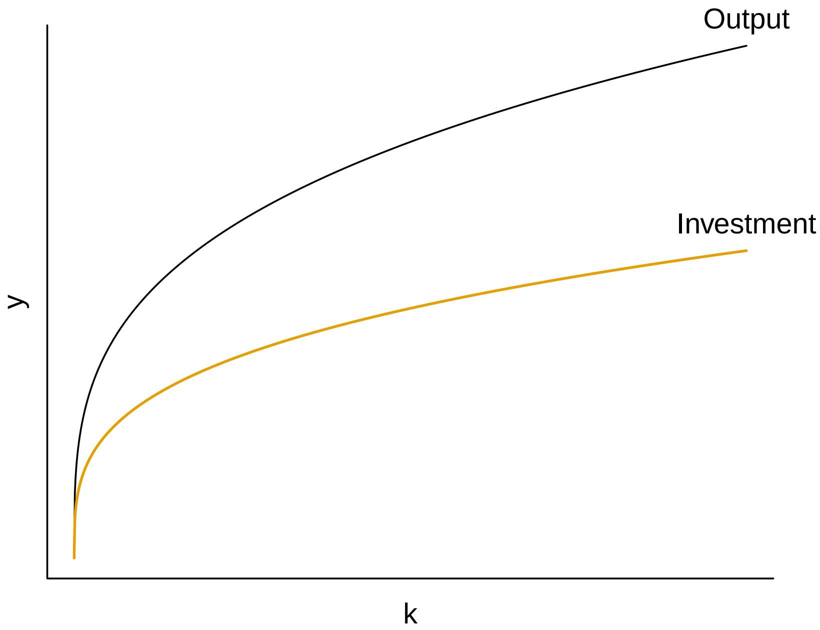
6 and 7 imply

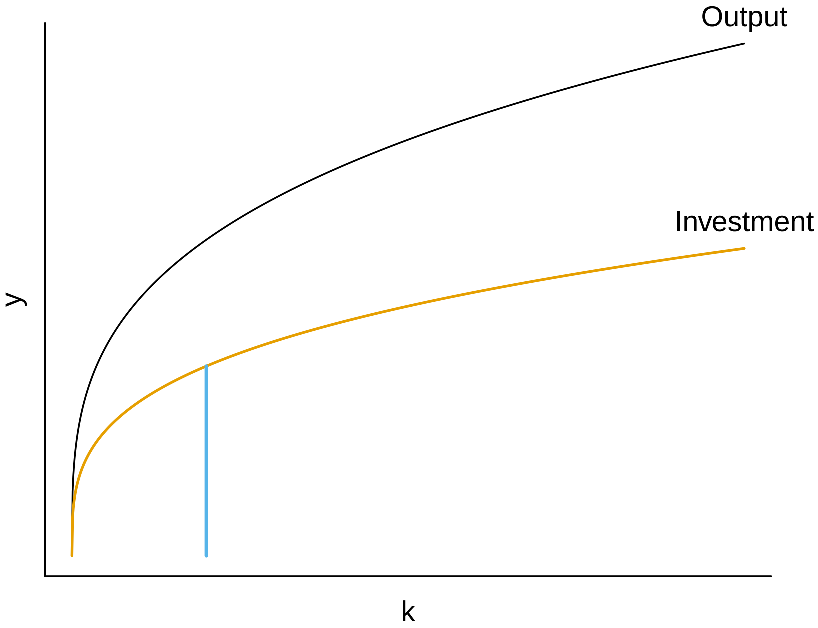
$$y = (1 - s)\, y + i \quad \Rightarrow \quad i = sy \tag{8}$$

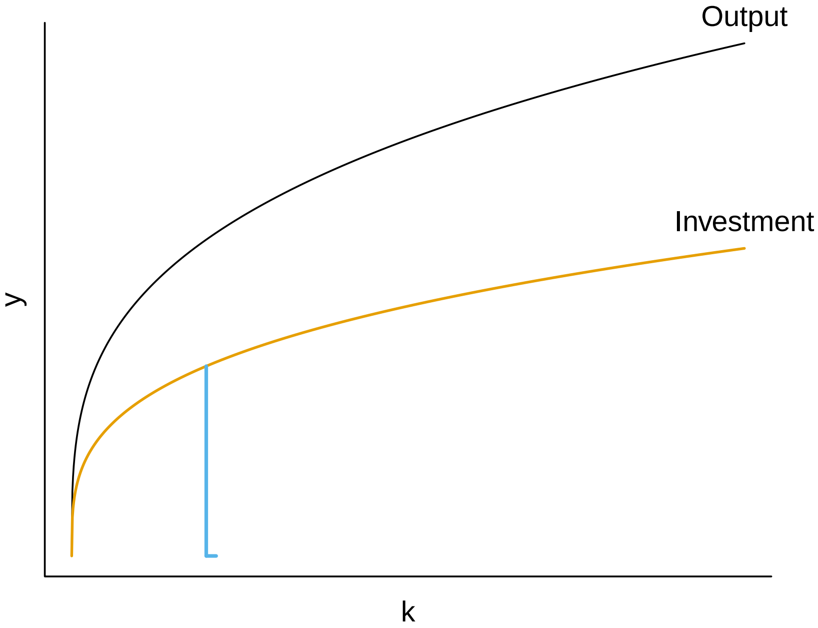
$y = c + i$ and $c = (1 - s)y$

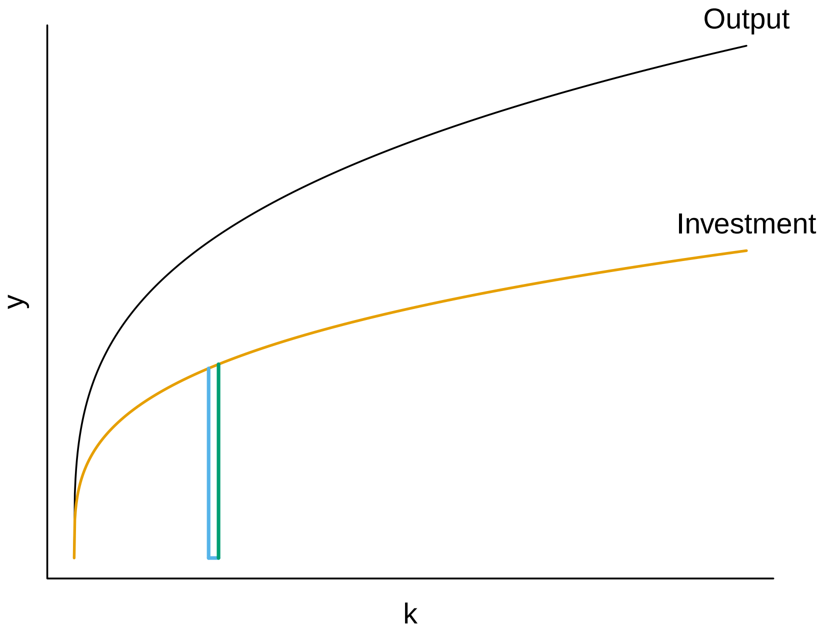


Output
per worker, y
Output, f(k)
Output
per worker
c
Consumption
per worker
y
Investment, sf(k)
i
Investment
per worker
Capital
per worker, k


Output
Investment
y
k


Output
Investment
y
k


Output
Investment
y
k


Output
Investment
y
k

What happens in the long-run?

- Grow forever
- Grow more slowly over time

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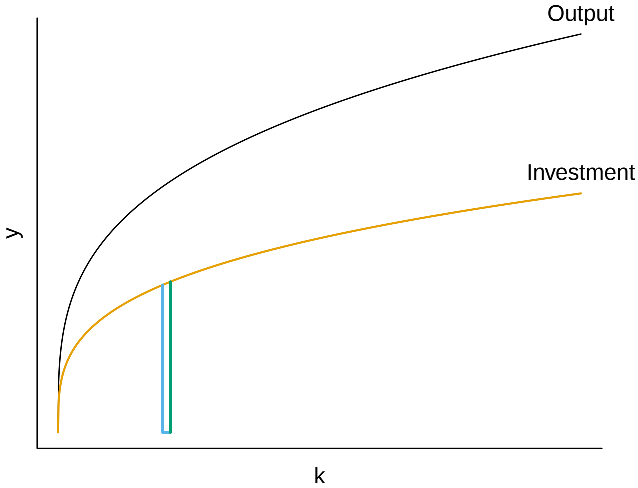
- Grow forever
- Grow more slowly over time

Is this a good model? What questions does it answer?

Which countries grow fastest?

High s
Low k



Problems with the model

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1. Do not observe continuous growth slowing down over time

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2. Capital breaks down

Depreciation

$$\text{Depreciation} = \delta k \tag{9}$$

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What is a reasonable number for δ?

Depreciation

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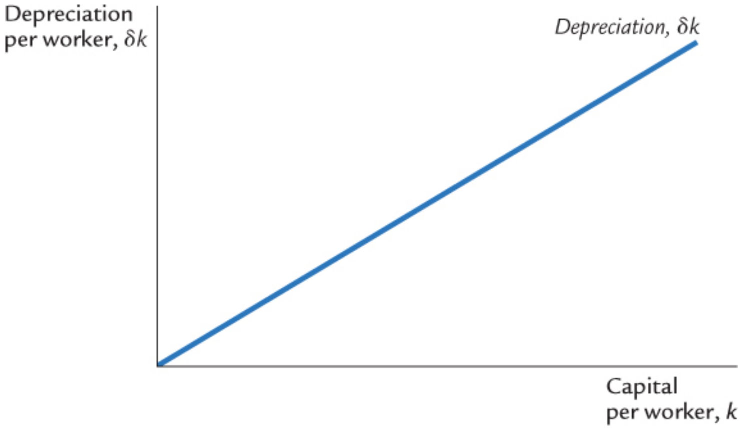
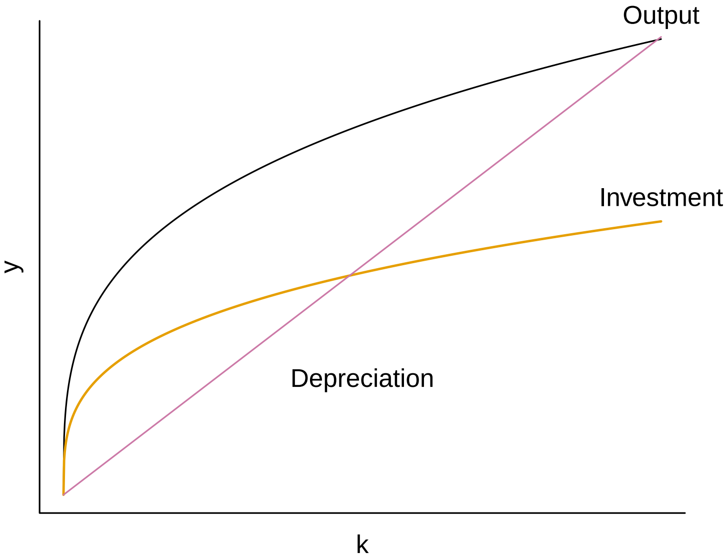
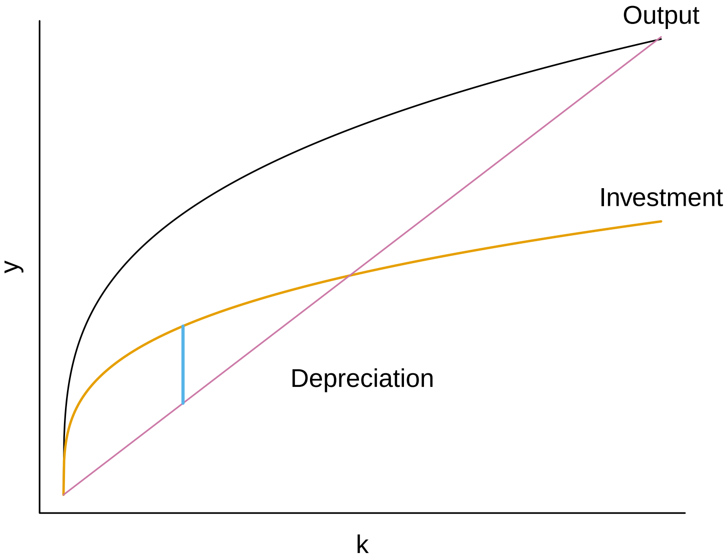
What is a reasonable number for δ?

If you buy machines that last 10 years on average, what is δ?


Depreciation
per worker, δk
Depreciation, δk
Capital
per worker, k


Output
Investment
Depreciation
y
k

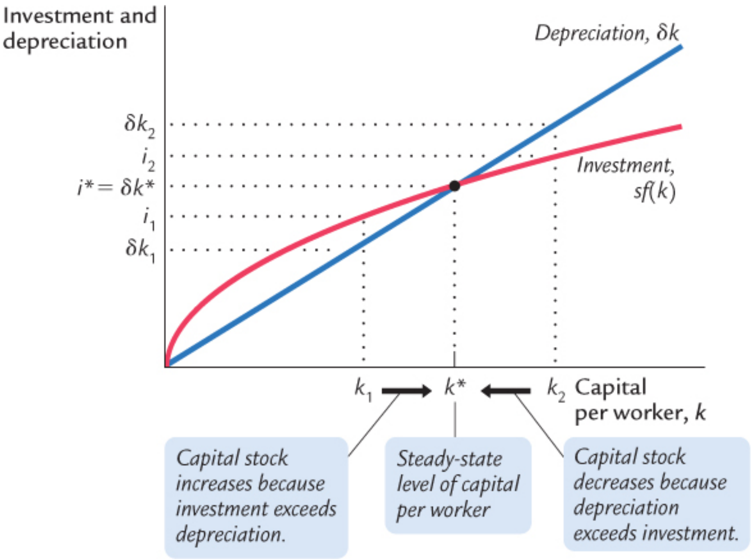

Output
Investment
y
Depreciation
k

What happens in the long-run?

What happens in the long-run?

No long-run growth

No long-run growth



Characterizing the steady state

$$\Delta k = \text{Investment} - \text{Depreciation} = sf(k) - \delta k \qquad (10)$$

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Characterizing the steady state

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$$\Rightarrow \delta k_{ss} = sf\left(k_{ss}\right) \tag{12}$$

$$\Rightarrow \frac{k_{ss}}{f\left(k_{ss}\right)} = \frac{s}{\delta} \tag{13}$$

Steady State: Cobb-Douglas

$$\begin{cases} Y &= K^{\frac{1}{3}} L^{\frac{2}{3}} \\ \delta &= .1 \\ s &= .2 \end{cases} \tag{14}$$

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4. Solve for k_{ss}

Suppose we develop a new technology to make our capital last longer

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- What happens to income growth rates?
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Remember our process:

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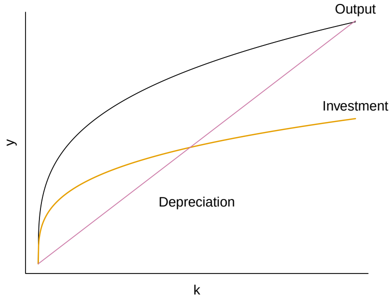
2. Shift curves
3. Find the new equilibrium
4. Figure out how we get there

We find ways to make our capital last longer



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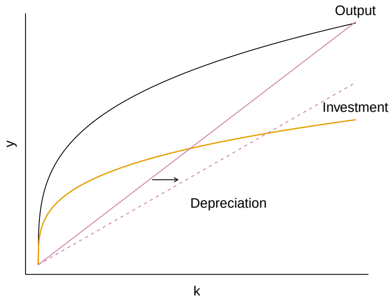

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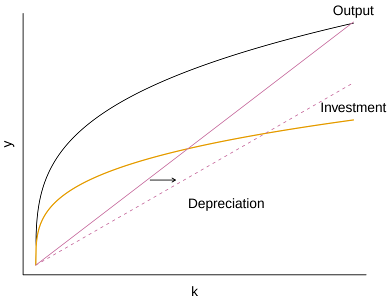

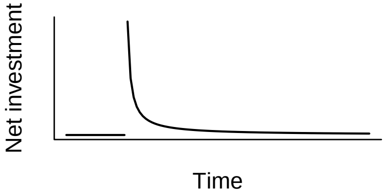

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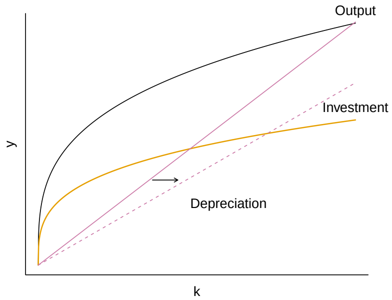
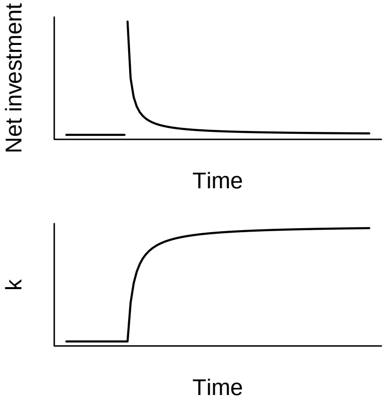
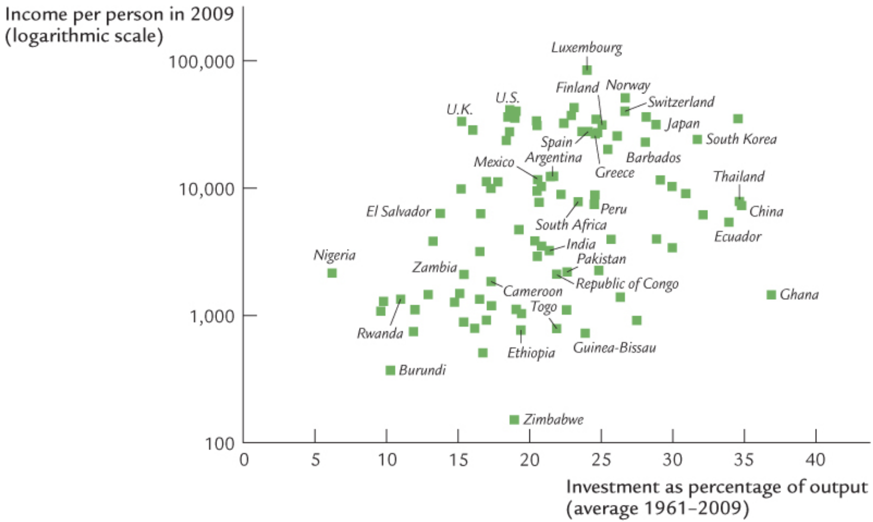




Are the predictions correct?



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Poopulation Growth

Produces long-run GDP growth

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Suppose L grows at rate n.

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Affects Δk

Suppose the population doubles every year. How fast do you need to build computers so that you always have one computer per person?

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$$\Delta k = i - (\delta + n)\, k \tag{15}$$

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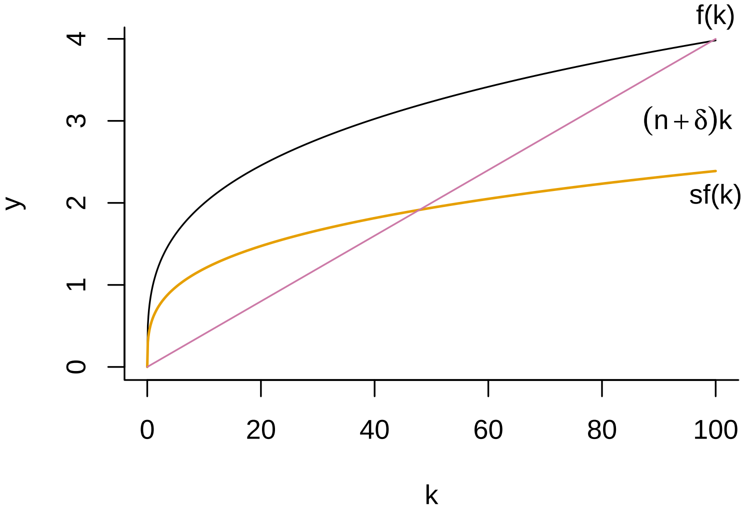
$$\Delta k = i - (\delta + n) k \tag{15}$$

$$\Rightarrow \Delta k = sf(k) - (\delta + n) k \tag{16}$$

Solow Model with Population Growth

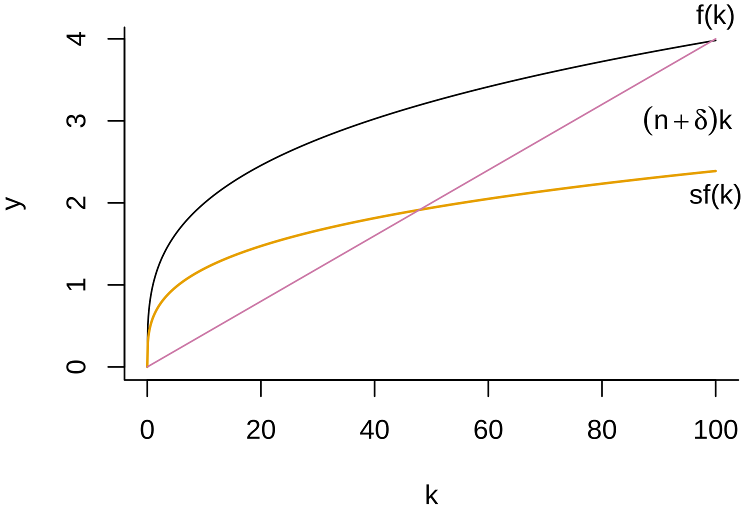


Steady State with Population Growth

$$sf(k_{ss}) = (\delta + n) k_{ss} \tag{17}$$

$$\Rightarrow \frac{k_{ss}}{f(k_{ss})} = \frac{s}{\delta + n} \tag{18}$$

What if n increases?



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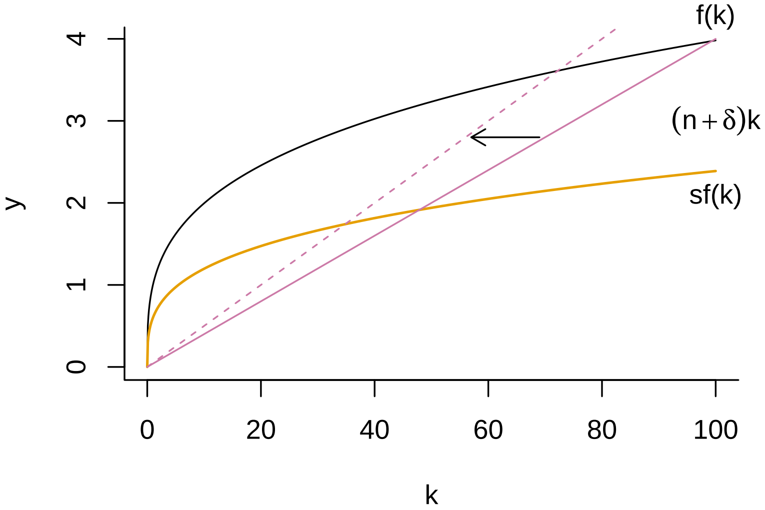
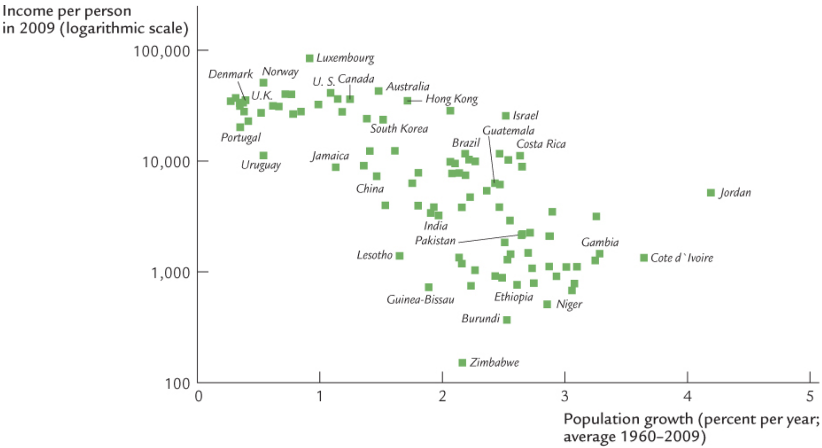


Are the Predictions Correct?

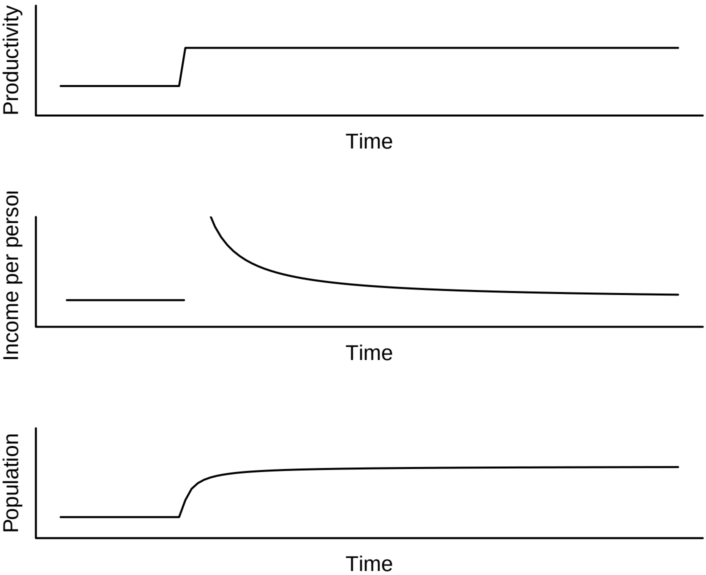


Malthusian Model

Historic argument (1800) that is very popular among lay audiences/press:

1. "Food is necessary for the existence of man."
2. "The passion between the sexes... will remain nearly in its present state."
3. Thus "the power of population is infinitely greater than the power of the earth to produce subsistence for man."

Malthusian trap: eternal poverty



Was Malthus right?

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Was Malthus right?

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What were his mistakes?

1. Applied a model of human fertility that is no longer relevant
2. Did not notice that people value things other than the number of children
3. Condoms

What happens to economic growth rates?
Capital? Incomes? Investment? Steady state?

1. A civil war destroys many of our factories.
2. People start saving more.
3. We find ways to make our capital last longer.

Remember our process:

1. Characterize the current situation
2. Shift curves
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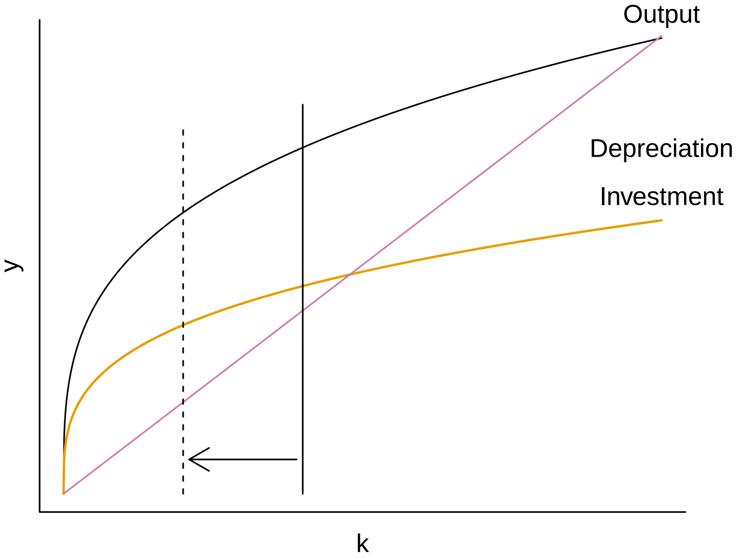
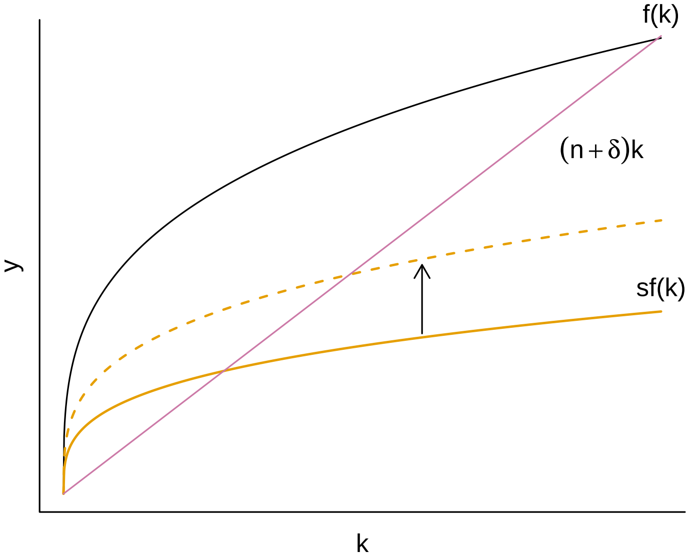
4. Figure out how we get there

A civil war destroys many of our factories



People start saving more



We find ways to make our capital last longer