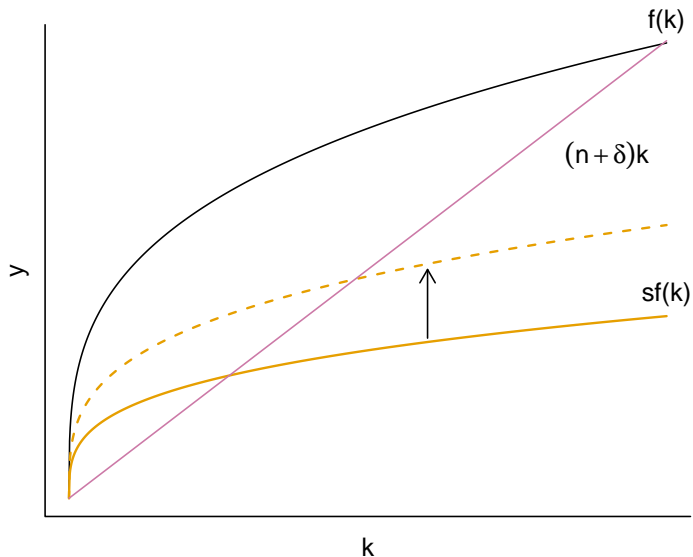


Saving's effect on consumption

How does s affect the steady state?

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The Golden Rule

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“Golden Rule”: choose s to maximize c

The Golden Rule

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Put these together

$$c_{ss} = f(k_{ss}) - (n + \delta) k_{ss} \quad (4)$$

The Golden Rule

What is k_{SS} ?

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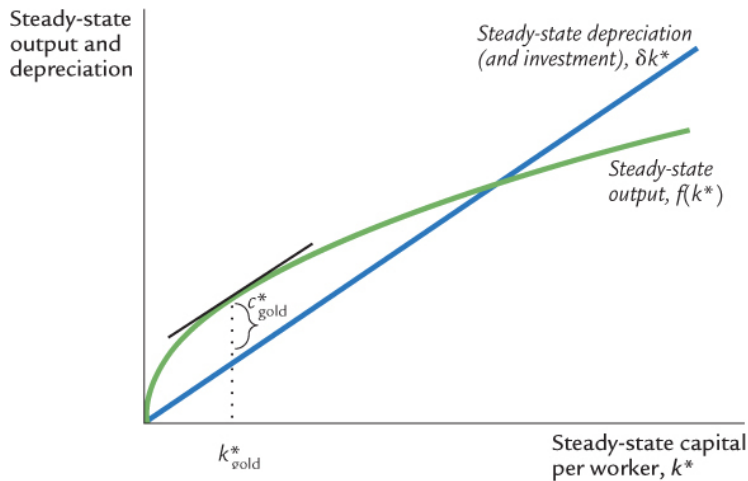
Higher $s \implies$ higher k_{SS}

The Golden Rule: Which SS is Best?

$$c_{SS} = f(k_{SS}) - (n + \delta)k_{SS}$$

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At the GR SS capital, the slopes are equal: $MPK_{gold} = n + \delta$

Golden Rule example

Model from yesterday:

$$y = k^{\frac{1}{3}} \quad (6)$$

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$$\Rightarrow k_{ss} = [3(n + \delta)]^{-\frac{3}{2}} \quad (10)$$

Golden Rule Savings

But how do we get to the GR capital stock?

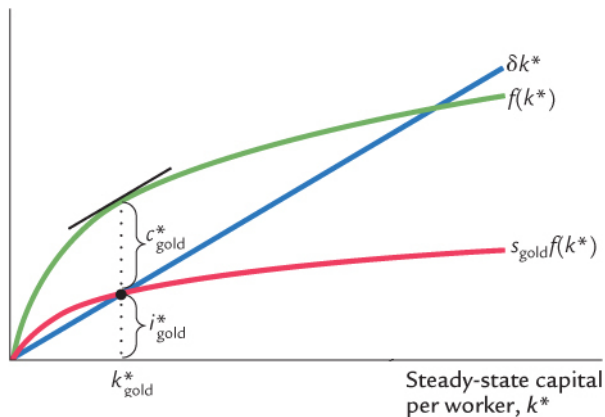
Golden Rule Savings

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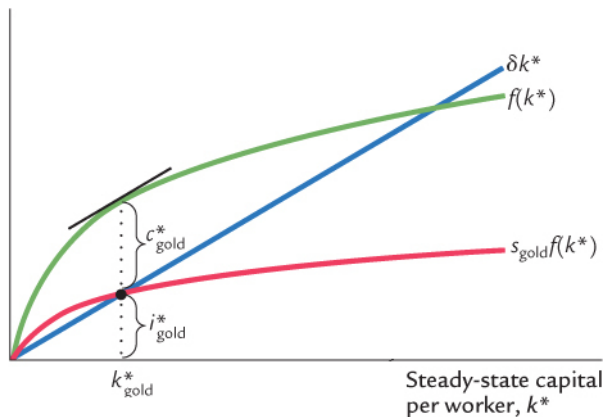
Steady-state output,
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investment per worker



Golden Rule Savings

But how do we get to the GR capital stock? Pick the right s .

Steady-state output,
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So why do the US and Japan pick different s ?

Back to our Example

What s do we need for the GR?

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From yesterday:

$$\frac{k_{SS}}{f(k_{SS})} = \frac{k_{SS}}{k_{SS}^{\frac{1}{3}}} = k_{SS}^{\frac{2}{3}} = \frac{s}{.1} \quad (13)$$

$$\Rightarrow .1 [3(n + \delta)]^{-1} = .1 (.3)^{-1} = \frac{1}{3} = s \quad (14)$$

The Golden Rule in Real Life

How do we know if a country is at the GR s or k_{SS} ?

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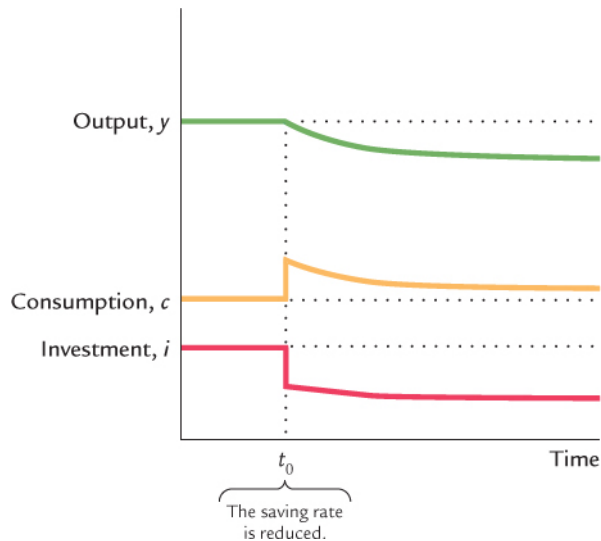
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How does s in the US compare to s_{GOLD} ?

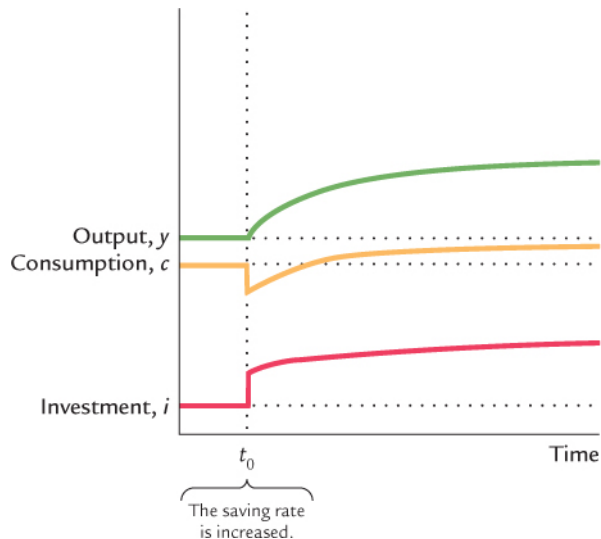
Transition to the GR: from too much k

Transition to the GR: from too much k



Transition to the GR: from too little k

Transition to the GR: from too little k



Should we move to the Golden Rule?

Who is hurt by the transition?

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Who is hurt by the transition?

Would you rather have \$100 today or in a year?

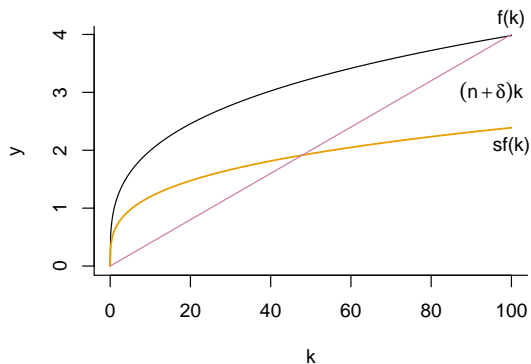
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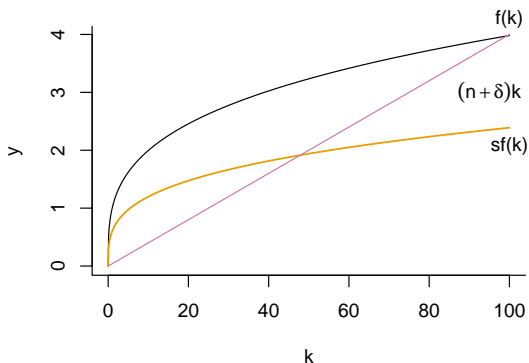
Should we expect to see countries above or below the GR?

Predictions of the Solow Model



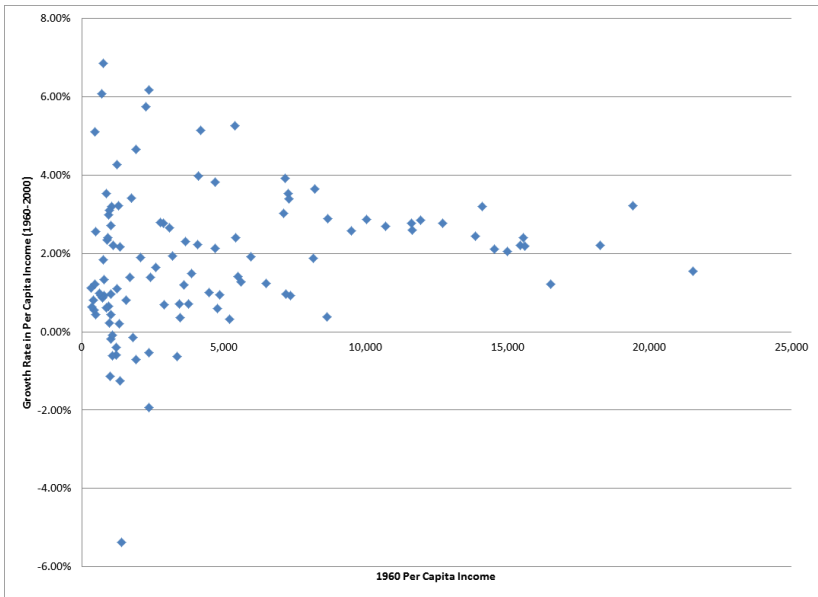
If one country is rich and one poor, which will grow faster?

Predictions of the Solow Model

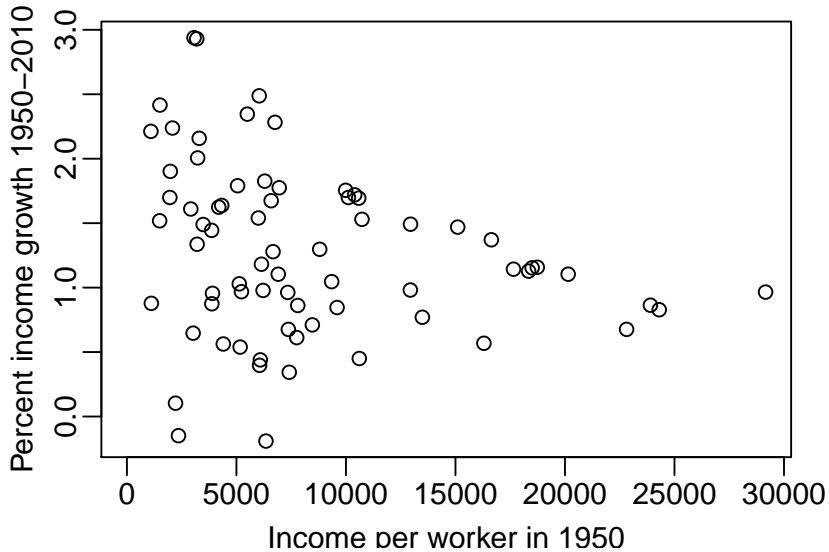


If one country is rich and one poor, which will grow faster?

Suggests that incomes around the world should converge to similar levels over time?



Conditional Convergence



Growth and per capita income, all countries, 2003--07



Does the Solow Model Predict Absolute Convergence?

Absolute convergence: poorer countries' incomes will grow faster.

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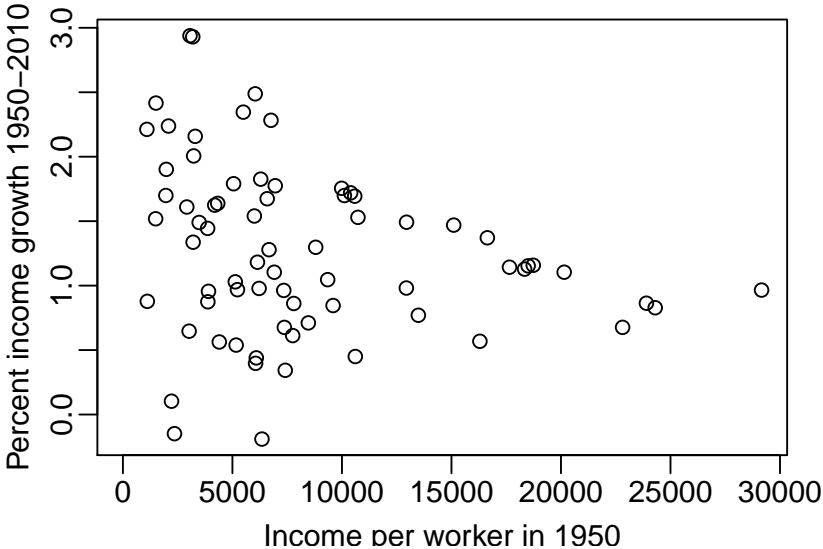
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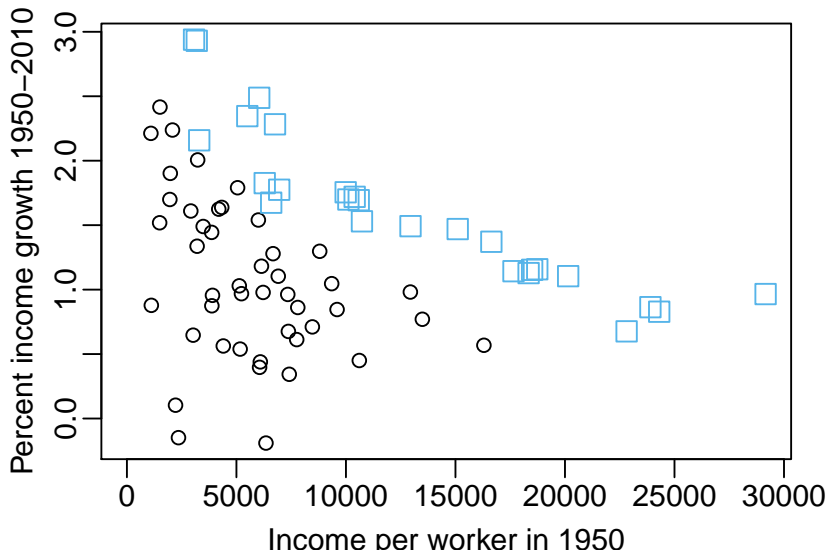
What if countries' saving rates differ?

What if their production functions differ?

Conditional Convergence



Conditional Convergence



How do we measure convergence?

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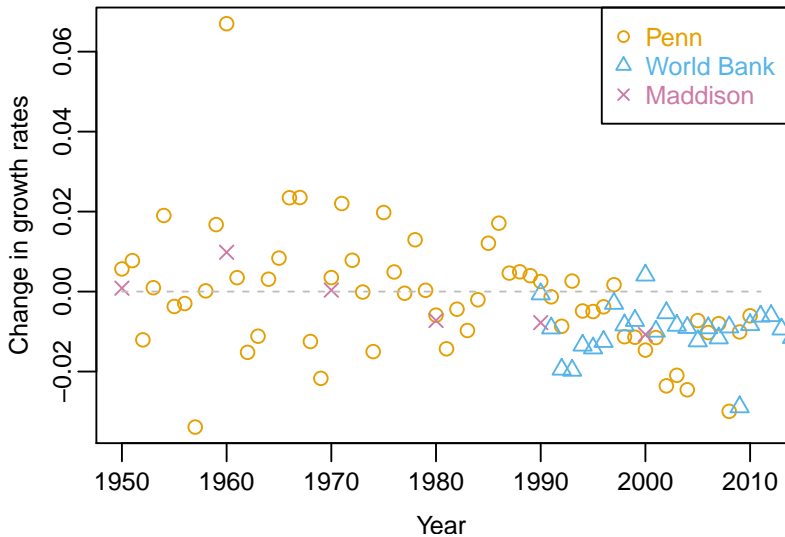
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- ▶ ϵ is a “disturbance term”

Increase in income growth rates from approximately doubling income



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- ▶ We do not know why production functions differ

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Solve both problems with **technology**

Technology

Technology: how we organize inputs to make output

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We get better at this over time

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Determines how efficient (productive) our inputs are

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Simple Solow Model with Technical Change

$$Y = F(K, E \times L) \quad (17)$$

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E : Efficiency of labor

$E \times L$: Effective number of workers

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Suppose E grows at rate g

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E : Efficiency of labor

$E \times L$: Effective number of workers

Suppose E grows at rate g

Then $E \times L$ grows at rate $n + g$

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$$\text{CRS} \implies \frac{Y}{E \times L} = F\left(\frac{K}{E \times L}, 1\right) \quad (18)$$

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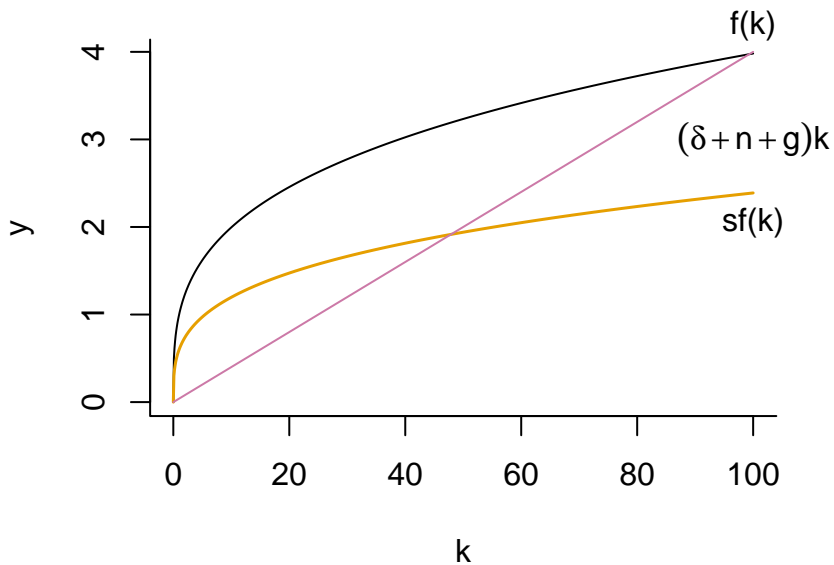
Rewrite this as

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Now it looks exactly the same as before, so we do the same thing

Simple Solow Model with Technical Change

$$\Delta \tilde{k} = sf(\tilde{k}) - (\delta + n + g) \tilde{k} \quad (20)$$



Simple Solow Model with Technical Change

What is constant in the SS?

Simple Solow Model with Technical Change

What is constant in the SS?

- ▶ Capital per effective worker

Simple Solow Model with Technical Change

What is constant in the SS?

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$$Y = \tilde{y} \times E \times L \quad (23)$$

Must be growing at rate $n + g$

Simple Solow Model with Technical Change

Total income grows in the long-run

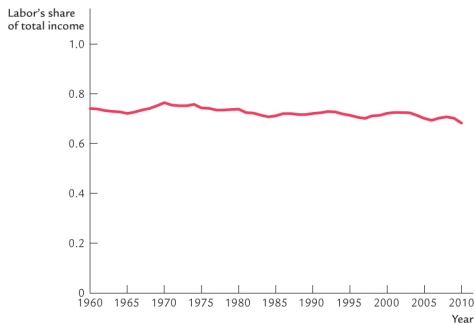
Individual incomes grow in the long-run

The capital stock grows in the long-run

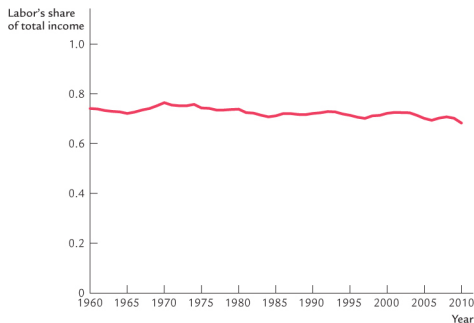
Success!

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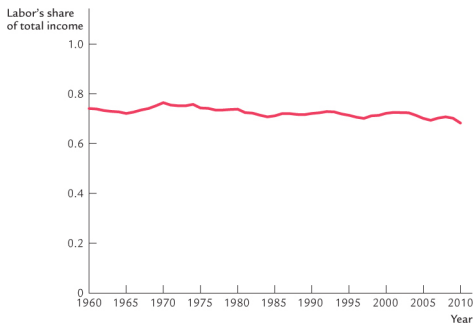


Cobb-Douglas production function \implies proportion of income going to capital is $r \frac{K}{Y}$



Showed that the income shares are constant

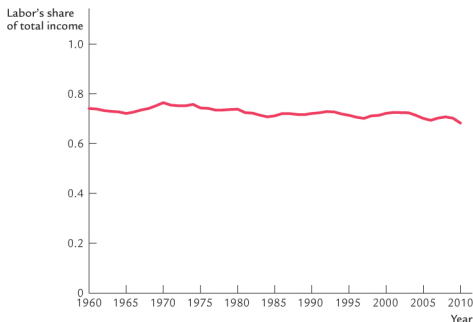
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So K and Y must grow at the same rate

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Showed that the income shares are constant

So K and Y must grow at the same rate

Success!

An alternative Solow Model with Tech

$$Y = AF(K, L) \implies y = Af(k) \quad (24)$$

